Appendix C

Additional Explanation of Multivariate Analysis

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Fixed Effects Model

The fixed effects model in equation (6) in the text could be expressed more fully by substituting equation (1) into equation (3), yielding:

$$ERROR_{p,it} + l\,ERROR_{n,it} = a_i + t_i'd_i + EFFORT'_{it}b_1 + PEFFORT_{it}'b_2 + X_{it}'g + e_{it} \qquad \text{(C-1)}$$

where the parameter λ was estimated via grid search by maximizing the log-likelihood function. The estimated value of λ was estimated at 1.45 in the fixed effects model. This estimate was robust to changes in the specification of the model, i.e., the inclusion or exclusion of variables and changes in functional form.

We optimized λ by choosing a value for λ, computing a new error measure (equal to positive error +(λ*negative error)), running a regression with the new error measure, and retaining both the value for λ and the log-likelihood. The regression with the largest log-likelihood indicated which value for λ was best. The lower bound for λ was -1, and the upper bound was 1.

Prais-Winsten FGLS Model

Allowing for first-order autocorrelation and heteroskedasticity in the fixed effects model described in (C-1), the residuals are given by:

$$e_{it} = r e_{it-1} + e_{it} \qquad \text{(C-2)}$$

$$e_{it} \sim N(0, s_i^2) \qquad \text{(C-3)}$$

where ρ is the autocorrelation coefficient.

For periods $t > 1$, the Prais-Winsten model is expressed as:

$$ERROR_{it}* = a_i* + t_i*'d_i + EFFORT_{it}*'b_1 + PEFFORT_{it}*'b_2 + X_{it}*'g + e_{it}* \qquad \text{(C-4)}$$

where the asterisks on each of the independent variables and the dependent variable denote the transformation given by:

$$V*_t = V_t - \rho V_{t-1} \qquad \text{(C-5)}$$

and the error structure is given by:

$$e*_{it} = \varepsilon_{it} \qquad \text{(C-6)}$$

Note that equation (C-5) requires subtracting a weighted lagged-value of an observation from that same variable's current period value. This cannot be done for the first observation, which might be discarded from the estimation, as in the Cochrane-Orcutt method. Prais-Winsten provide an alternative transformation for the first time period information. We employ the Prais-Winsten method of weighting the first-year's observations by $(1-\rho^2)^{1/2}$. Thus, we rewrite equation (C-5) for $t = 1$ as:

$$V^*_1 = (1-\rho^2)^{1/2} V_1 \tag{C-7}$$

and the error term in period 1 is given by:

$$e^*_{it} = (1-\rho^2)^{1/2} \varepsilon_{it} \tag{C-8}$$

After transforming the data, the regression for estimation can be written for all t as:

$$ERROR_{it}^* = \alpha_i^* + t_i^{*\prime}\delta_i + EFFORT_{it}^{*\prime}\beta_1 + PEFFORT_{it}^{*\prime}\beta_2 + X_{it}^{*\prime}\gamma + \varepsilon_{it}^* \tag{C-9}$$

The model described by equation (C-9) is a special case of a more general model where the autocorrelation is expressed as:

$$e_{it} = \rho_i e_{it-1} + \varepsilon_{it} \tag{C-10}$$

where ρ is subscripted by state i. That is, state-specific autocorrelation parameters are estimated, allowing there to be differences in autocorrelation across states. Equation (C-9) is then estimated as above, using a state-specific autocorrelation parameter.

Relative to the Prais-Winsten model using a common ρ, the model using a state-specific estimate of ρ may produce less biased estimates if the autocorrelation parameters are not equal across states. It may be less efficient, however, because it requires additional parameter estimates. Thus, the estimates using a common ρ will be consistent and efficient if the autocorrelation coefficient does not vary across states, while the estimates using state-specific values of ρ will be consistent when the autocorrelation coefficient does vary across states, but will be inefficient if it does not.

Partial Adjustment Model

The partial adjustment model assumes that states adjust their resources so as to achieve a desired level of errors, but only make these adjustments gradually. That is, we assume:

$$ERROR_{it} - ERROR_{it-1} = (1-\psi)\left(ERROR^*_{it} - ERROR_{it-1}\right) \tag{C-11}$$

where ψ is the fraction of the gap that is closed within a year and $ERROR_{it}^*$ is the desired error rate of state i at time t. Then, rewriting equation (6) in the text as the target level of ERROR, we have:

$$ERROR^*_{it} = \alpha_i + t_i'\delta_i + EFFORT'_{it}\beta_1 + PEFFORT_{it}'\beta_2 + X_{it}'\gamma + \varepsilon_{it} \tag{C-12}$$

Because we cannot observe the targeted level of error, however, we substitute equation (C-11) into (C-12) and solve for the observed error rate:

$$ERROR_{it} = \tilde{\alpha}_i + t_i'\tilde{\delta}_i + \psi e_{it-1} + EFFORT'_{it}\tilde{\beta}_1 + PEFFORT_{it}'\tilde{\beta}_2 + X_{it}'\tilde{\gamma} + \nu_{it} \tag{C-13}$$

where the above coefficients with the tildes (such as $\tilde{b}_1$) relate to the original coefficients in equation (6) in the text by a factor of (1/1-ψ), with $|\psi|<1$. The long-run effect of EFFORT in the pre-PRWORA period is then given by the following relationship:

$$b_1 = \frac{\tilde{b}_1}{1-y} \tag{C-14}$$

The variances for the long-run estimates are calculated via the delta method. Using a linear expansion, $Var(b_1)$ is given as dVd’ where:

$$d \approx \left[\frac{1}{1-y} \quad \frac{\tilde{b}_1}{(1-y)^2} \right] \tag{C-15}$$

is a row vector, approximated with estimates of ψ and $\tilde{b}_1$, and V is a 2×2 matrix whose elements are the estimated sampling variances and covariances for $\tilde{b}_1$ and $\hat{y}$. The calculation of parameter estimates and sampling variances of long-run effects of other covariates are analogous to that described in equation (C-14) and equation (C-15) above.

Arellano-Bond Model

The Arellano-Bond model is based on a method of instrumental variables to surmount the problem of bias and inconsistency introduced when using the lagged dependent variable as a regressor. The model is based on equation (C-13). The disturbances, ν_{it}, are assumed to have finite moments with $E(\nu_{it}) = E(\nu_{is}\nu_{it}) = 0$ for $s \neq t$. This assumption assumes that there is no serial correlation, but does not require independence over time.

Under these assumptions, values of the dependent variable, ERROR, lagged two periods can be used as valid instruments. For simplicity, we re-write equation (C-13) as:

$$ERROR_{it} = \tilde{a}_i + y\,ERROR_{it-1} + W'_{it}p + n_{it} \tag{C-16}$$

The equation in (C-16) is then first-differenced, thus removing $\tilde{a}_i$ and producing an equation that is estimable via instrumental variables, using two-period lagged values of $ERROR_{it}$. Arellano and Bond (1991) note that for panels with at least three time periods, the model implies m = (T-2)(T-1)/2 linear moment restrictions:

$$E\left[\left(\overline{ERROR}_{it} - y\,\overline{ERROR}_{it-1} - \overline{W}'_{it}p\right)v_{it-j}\right] = 0 \qquad j = 2,...,(t-1); \quad t = 3,....,T \tag{C-17}$$

where $\overline{ERROR}_{it} = ERROR_{it} - ERROR_{it-1}$. The estimates of the coefficients in (C-16) are obtained via generalized methods of moments (GMM). For further simplicity, including the lagged values of $ERROR_{it}$ as instruments, we rewrite equation (C-16) as:

$$ERROR_{it} = K'_{it}\kappa + \nu_{it} \qquad \text{(C-18)}$$

Then, following Arellano and Bond (1991), the GMM estimator $\hat{\kappa}$ is given by the following kx1 coefficient vector:

$$\hat{\kappa} = \left(\vec{K}' Z A_N Z' \vec{K}\right)^{-1} \vec{K}' Z A_N Z' \vec{e} \qquad \text{(C-19)}$$

where $\vec{K}$ is a stacked (T-2)Nxk matrix of observations on $\overline{ERROR}$, Z_i = diag ($ERROR_{i1}$,…,$ERROR_{is}$, K_{i1},….,K_{is}) for *s*=1,…,T-2, and A_N is given by V^{-1}_N, where:

$$\hat{V}_N = N^{-1} \sum_i Z_i' \hat{\nu}_i \hat{\nu}_i' Z_i \qquad \text{(C-20)}$$

The long-run estimate of the effect of effort on error is computed analogously to equation (C-14) above, where ψ is now the parameter estimate associated with the instrument. The standard error associated with the long-run estimate is calculated via the delta method, analogously to equation (C-15).

Elasticities

In the pre-PRWORA period, the effort elasticity, η_{PRE}, is calculated as:

$$\eta_{PRE} = \left[\frac{\partial\, ERROR}{\partial\, EFFORT}\right]\left[\frac{\overline{EFFORT}_{PRE}}{\overline{ERROR}_{PRE}}\right] = \beta_1 \left[\frac{\overline{EFFORT}_{PRE}}{\overline{ERROR}_{PRE}}\right] \qquad \text{(C-21)}$$

Note that for the simple partial adjustment model and the Arellano-Bond model, we use the long-run estimate of the effect of effort on error rates so as to make the elasticities comparable across models. The variance of the pre-PRWORA elasticity is then given by:

$$Var(\eta_{PRE}) = \left[\frac{\overline{EFFORT}_{PRE}}{\overline{ERROR}_{PRE}}\right]^2 Var(\beta_1) \qquad \text{(C-22)}$$

Note again that calculating Var (β_1) for the partial adjustment and Arellano-Bond models requires the approximation described by equation (C-15).

For the post-PRWORA period, elasticity calculations become slightly more complicated because of the inclusion of the PEFFORT variable in the model. The post-PRWORA effort elasticity, η_{POST}, is calculated as:

$$\eta_{POST} = \left[\left(\frac{\partial\, ERROR}{\partial\, EFFORT} + \frac{\partial\, ERROR}{\partial\, PEFFORT}\right)\right]\left[\frac{\overline{EFFORT}_{POST}}{\overline{ERROR}_{POST}}\right] = (\beta_1 + \beta_2)\left[\frac{\overline{EFFORT}_{POST}}{\overline{ERROR}_{POST}}\right] \qquad \text{(C-23)}$$

where, again, for the partial adjustment model and the Arellano-Bond model, the long-run estimates of the βs are used. The calculation of the variances of the elasticities differs slightly across the models. For the fixed effects and Prais-Winsten FGLS models, the variance of the post-PRWORA effort elasticity is given by:

$$Var\,(h_{POST}) = \left[\frac{\overline{EFFORT}_{POST}}{\overline{ERROR}_{POST}}\right]^2 \left[Var(b_1) + Var(b_2) + 2Cov(b_1 b_2)\right] \qquad \text{(C-24)}$$

For the partial adjustment model and the Arellano-Bond model, we must calculate:

$$Var\,(h_{POST}) = \left[\frac{\overline{EFFORT}_{POST}}{\overline{ERROR}_{POST}}\right]^2 Var\left(\frac{\tilde{b}_1}{1-y} + \frac{\tilde{b}_2}{1-y}\right) \qquad \text{(C-25)}$$

The second term in equation (C-25) is approximated as V'CV, where V is a column vector containing three elements:

$$V = \begin{bmatrix} \dfrac{1}{1-y} \\ \dfrac{1}{1-y} \\ \dfrac{\tilde{b}_1 + \tilde{b}_2}{(1-y)^2} \end{bmatrix} \qquad \text{(C-26)}$$

and C is the 3x3 variance-covariance matrix of the three elements, $\tilde{b}_1, \tilde{b}_2,$ and ψ. Thus, the diagonal is given by $Var(\tilde{b}_1), Var(\tilde{b}_2),$ and $Var(y)$.